Final Spin from Binary Black Hole Coalescence

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Numerical relativity

- We are interested to accurately determine the gravitational wave content and physical properties of spacetimes which are:
  - Strong.
  - Dynamical.
  - Without symmetries.
- In the strong-field, dynamical regime, nonlinear terms of the Einstein equation play an important role – approximations break down.
- We resort to numerical computation (computer simulation) to determine solutions of the Einstein equations:

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi T_{\alpha\beta}$$

- For the purpose of this talk, we consider only vacuum solutions, ie. $T_{\alpha\beta} = 0$.

Cactus

- BSSN 1st order in space, 2nd order in time
- 1+log lapse
  For the Bona-Massó slicing conditions:
  $$(\partial_t - \beta^i \partial_i)\alpha = -\alpha^2 f(\alpha) K$$
  we set $f = 2/\alpha$
- Γ-driver shift evolution
  $$\partial_t \beta^i = k \partial_t \tilde{\Gamma}^i \qquad (k > 0)$$
- Carpet Adaptive Mesh Refinement follows Puncture movement
- Wave Extraction both with Zerilli Extraction and the Newman-Penrose Ψ_4
- Puncture initial data with PN derived orbital parameters
  Assume a conformal factor of the form:
  $$\psi = u + \Sigma_{i=1}^{N} \frac{m_i}{2r_i}.$$
  Find C^2 solutions for u of the Hamiltonian constraint:
  $$\tilde{\nabla}^2 u + \frac{1}{8}\chi^7 \tilde{A}_{ij}\tilde{A}^{ij}(1+\chi u)^{-7}) = 0$$

BH Astrophysics with Numerical Relativity

- There are a number of interesting physics results available from studying the last orbits, plunge and ringdown.
  - State of the final BH from generic initial data
  - Recoil of the final BH
  - Mode decomposition of the plunge waveform.
- These results are easily accessible, given reasonable quasi-circular/PN orbit parameters at late times.

In a series of papers, we have studied the merger physics of binary-BHs with spins:

- Rezzolla et al., "Spin Diagrams for Equal-Mass Black-Hole Binaries with Aligned Spins",
- Rezzolla et al., "The final spin from the coalescence of aligned-spin black-hole binaries",
- Rezzolla et al., "On the final spin from the coalescence of two black holes",

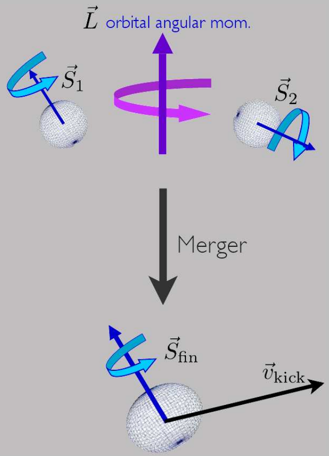
Black Hole Spins

- The objective of this talk is to derive a phenomenological formula for spin of a black hole resulting from the merger of two black holes of arbitrarily oriented spins and generic mass ratio
- This has applications for:
  - statistical distribution of black hole properties
  - simulations of the central regions of galaxies
  - dynamics of star clusters
- We need to simulate 2 spinning black holes over a 7D parameter space

$$\{S_1^i, S_2^i, M_1/M_2\}$$

to get one final black hole

$$\{v_{kick}^i\}\{S_{fin}^i/M_{fin}^2\}$$



Black Hole Spins

- We obtain a general 2nd order polynomial expansion with 5 restricting assumptions for our coefficients:
  - mass radiated in gravitational waves may be neglected, $M_{fin} \approx M$:

$$M_{rad}/M = 1 - M_{fin}/M \approx 5 - 7x10^{-2}$$

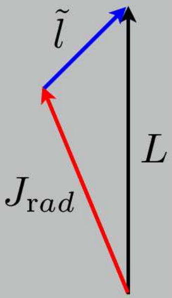
  - magnitude of the final spin vector is the sum of the initial spin vectors plus a third vector, $\tilde{\ell}$:

$$S_{fin} = S_1 + S_2 + \tilde{\ell}$$

$$\ell = L - J_{\mathrm{rad}}$$

  - The vector $\tilde{\ell}$ is parallel to L with a resulting error in the estimate of $\sim |J^{\perp}_{\mathrm{rad}}|^2/|\tilde{\ell}|^2 \sim |J^{\perp}_{\mathrm{rad}}|^2/(2\sqrt{3}M_1M_2)^2$ these errors are small in all the configurations that we have analysed
  - When the initial spin vectors are equal and opposite ($S_1 = -S_2$) and the masses are equal ($q = 1$), the spin of the final black hole is the same as for the non-spinning binaries
  - The extreme mass ratio limit (EMRL) is trivial

$$S_{fin} = S_1 \qquad if \qquad M \mapsto 0$$



Black Hole Spins

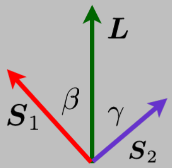
- Using these assumptions, it follows that:

$$|a_{\text{fin}}| = \frac{1}{(1+q)^2}\Big[|a_1|^2 + |a_2|^2 q^4 + 2|a_2||a_1|q^2\cos\alpha + 2\left(|a_1|\cos\beta + |a_2|q^2\cos\gamma\right)|\ell|q + |\ell|^2 q^2\Big]^{1/2},$$

  where $\cos\alpha \equiv \hat{a}_1 \cdot \hat{a}_2$, $\cos\beta \equiv \hat{a}_1 \cdot \hat{\ell}$, $\cos\gamma \equiv \hat{a}_2 \cdot \hat{\ell}$.
- In order to obtain $|\ell|$ we need to match this equation against general second order polynomial expansions for:
  - Equal mass, unequal but aligned spin binaries
  - Unequal mass, equal spin binaries

$$|\ell| = \frac{s_4}{(1+q^2)^2}\left(|a_1|^2 + |a_2|^2 q^4 + 2|a_1||a_2|q^2\cos\alpha\right) + \left(\frac{s_5\nu + t_0 + 2}{1+q^2}\right)\left(|a_1|\cos\beta + |a_2|q^2\cos\gamma\right) + 2\sqrt{3} + t_2\nu + t_3\nu^2 .$$

  - Numerical simulations to obtain s_4, s_5, t_0, t_2, t_3.
  - Test against generic misaligned spin binaries.



Final spin via horizon shape

- Valid once a common horizon has formed and settled down to a perturbed state.
- Measure equatorial circumference C_e and polar circumference C_p along orthogonal great circles.
- $C_r = C_p/C_e$ settles to a constant value:

$$C_r(j) = \frac{1+\sqrt{1-j^2}}{\pi} E\left(-\frac{j^2}{(1+\sqrt{1-j^2})^2}\right)$$

where $j = a/M$, and $E(k)$ is the complete elliptic integral of the second kind

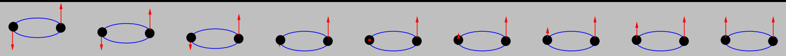
$$E(k) = \int_0^{\pi/2} \sqrt{1-k\sin^2\theta}\, d\theta.$$

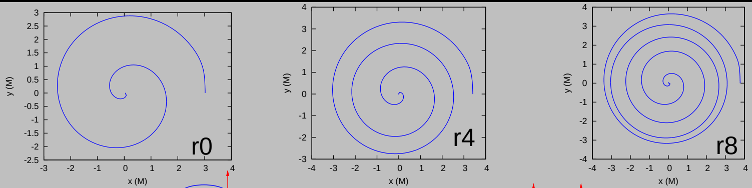
- This equation is integrated numerically to obtain j from the horizon shape.

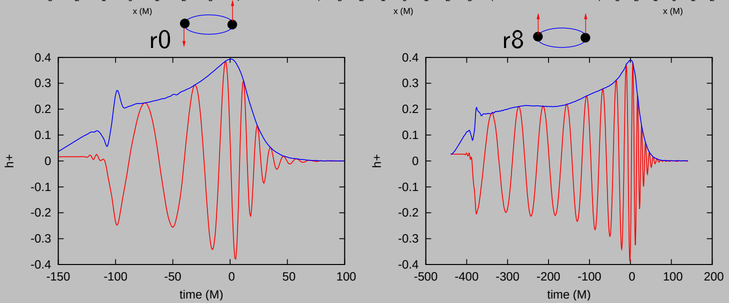
Parameter studies with spinning black holes

Aligned spin leads to an orbital hang-up.

r0	r1	r2	r3	r4	r5	r6	r7	r8

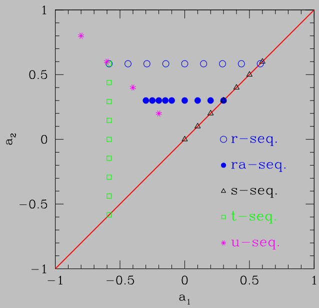


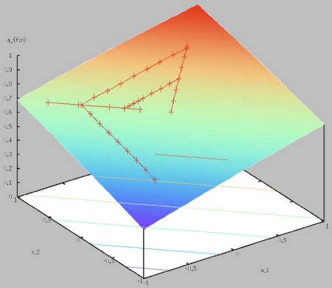




Equal Mass, Aligned Unequal Spin Binaries

- We have carried out studies in the parameter space of equal-mass aligned spin binaries, starting from non-eccentric orbit.
- Vary the spin of each BH from $a = -0.6$ to $a = +0.6$.
- Initial studies determined final BH parameters (final spin, radiated energy, kick) as a function of binary parameters.
- Kick depends quadratically on the spin difference, up to $\sim 450 km/s$ in the maximal case.
- Final spin is an almost linear function of the initial spins.





Spin of the final BH.

Aligned Unequal Spins, Equal Mass

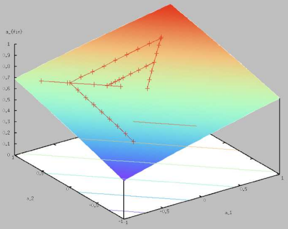
The resulting expression is:

$$a_{\text{fin}} = p_0 + p_1(a_1 + a_2) + p_2(a_1 + a_2)^2 \,.$$

with

$$p_0 = 0.6883 \pm 0.0003, \quad p_1 = 0.1530 \pm 0.0004, \quad p_2 = -0.0088 \pm 0.0005,$$

$$p_0 = \frac{\sqrt{3}}{2} + \frac{t_2}{16} + \frac{t_3}{64}, \quad p_1 = \frac{1}{2} + \frac{s_5}{32} + \frac{t_0}{8}, \quad p_2 = \frac{s_4}{16}.$$



- predicts a minimum and maximum spin:

  $$(a_{fin})_{min} \approx 0.347$$

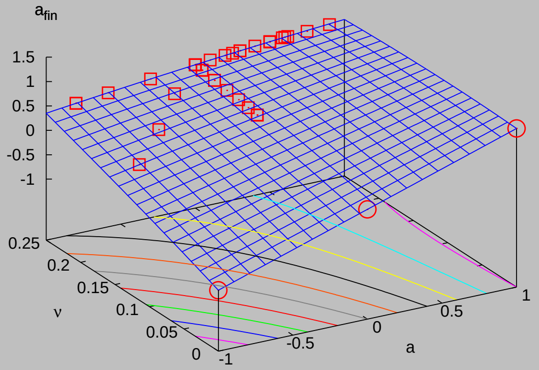
  $$(a_{fin})_{max} \approx 0.959$$

  for aligned spins.

Unequal Mass, Aligned Spins

- The spin of the final black hole has been determined for very generic initial conditions:
  - Arbitrary aligned spins
  - Unequal masses
- In the extreme-mass-ratio limit, approximation methods can be used.

$$a_{\mathrm{fin}} = a + s_4 a^2 \nu + s_5 a \nu^2 + t_0 a \nu + 2\sqrt{3}\nu + t_2 \nu^2 + t_3 \nu^3$$



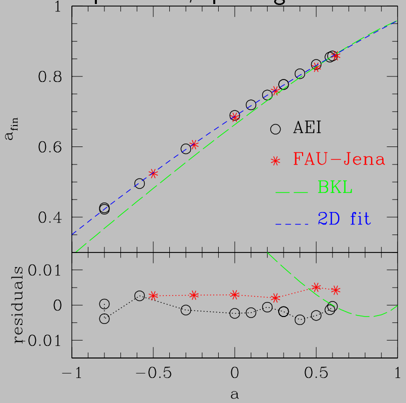
$$s_4 = -0.129 \pm 0.012$$
$$s_5 = -0.384 \pm 0.261$$
$$t_0 = -2.686 \pm 0.065$$
$$t_2 = -3.454 \pm 0.132$$
$$t_3 = 2.353 \pm 0.548$$

Accuracy for Aligned spins

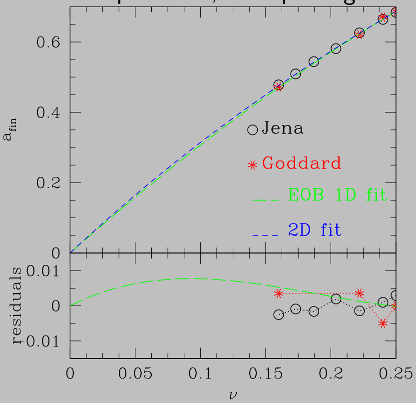
- Numerical relativity results for non-spinning BHs (Jena, Goddard, Penn State)
- Extreme mass ratio calculations for the $m_1 \gg m_2$ limit (Buonanno-Kidder-Lehner 2007)

$$a_{\rm fin} = a + s_4 a^2 \nu + s_5 a \nu^2 + t_0 a \nu + 2\sqrt{3}\nu + t_2 \nu^2 + t_3 \nu^3$$

Equal-mass, spinning

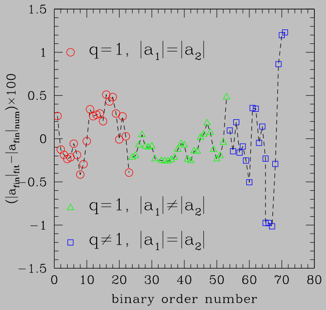


Unequal mass, non-spinning



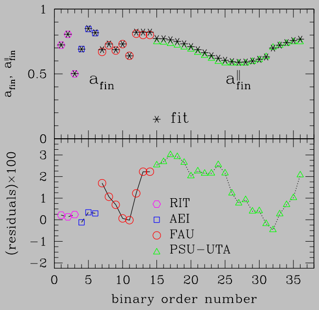
Accuracy for Misaligned Spins

Aligned spinning



- Circles refer to equal-mass, equal-spin binaries in (Rezzolla-etal, Marronetti-etal, Berti-etal, Buonanno-etal 2007),
- Triangles to equal-mass, unequal-spin binaries in (Rezzolla-etal, Berti-etal),
- Squares to unequal-mass, equal-spin binaries in (Berti-etal,Buonanno-etal,Rezzolla-etal).

Misaligned spinning



- Hexagons refer to data from ref. (Campanelli:2006vp)
- Squares to the data obtained in AEI runs
- Circles to data from ref. (Tichy:2007gso)
- Triangles to data from ref. (Herrmann:2007ex)

Summary

- Binary black holes are a fertile ground for gravitational physics (recoil, spin, waveforms).
  - Modelling of final spins and kicks within a few percent precision
  - Hybrid methods, combining post-Newtonian and perturbative approaches with numerical results are starting to provide a picture of the full inspiral-merger process.
- Techniques for numerical relativity are now rather advanced. There are still systematic problems to be tackled:
  - Efficiency.
  - Improving initial data construction.
  - Understanding limitations of wave extraction at a finite radius.
- Room for improvement by including J_{rad} in the orbital plane
- Interesting to see what we can discover about extremal spin regions