Binary Orbital Dynamics from the Analysis of Spherical Harmonic Modes of Gravitational Waves

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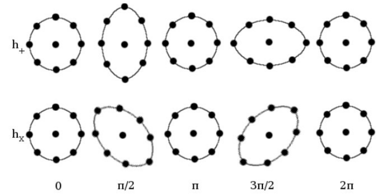
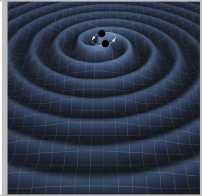
Outline

I show how to extract binary orbital dynamics from GWs by minimization of the asymmetric spherical harmonic modes of gravitational waveforms.

General Relativity and Gravitational Waves





- Coaction between matter and curvature is described by the Einstein Equations:

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

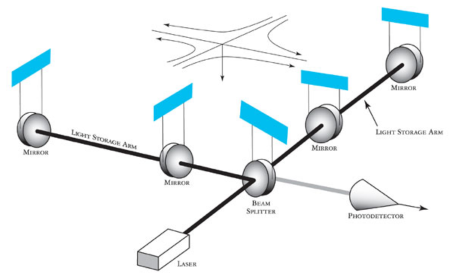
- Black holes (BH) = Vacuum ($T_{\mu\nu} = 0$)
- Gravitational Waves (GW) = finite deviation from Minkowski spacetime:

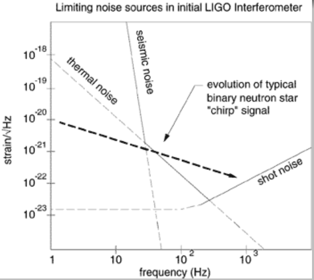
$$g_{\mu\nu} = \eta_{\nu\mu} + h_{\nu\mu}\,, \qquad |h_{\nu\mu}| \ll 1.$$

- Linearized field equations in GR

$$\Box \bar{h}_{\mu\nu} = 16\pi T_{\mu\nu} = (-\partial_t^2 + \nabla^2)\bar{h}_{\mu\nu} = 0.$$

Gravitational waves





- The coupling between matter and geometry is very weak.

$$R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = k T_{\alpha\beta}$$

$$k = \frac{8\pi G}{c^4} \simeq 2 \times 10^{-43} \frac{s^2}{m \cdot kg}$$

- Gravitational waves are small features, difficult to detect.
- Unobstructed by intervening matter
- Excellent probe into regions opaque to EM radiation.

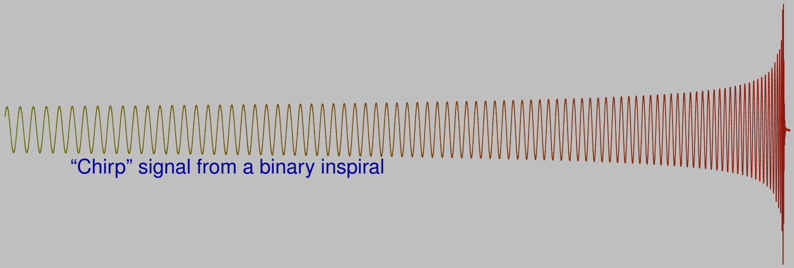
Gravitational waves

- Currently there are many ground based detectors online which are designed to detect such passing gravitational waves (LIGO, VIRGO, TAMA, GEO).
- Even for binary black hole inspiral and merger, the signal strength is likely to be much less than the level of any detector noise.
- A technique used for this purpose is *matched filtering*, in which the detector output is cross-correlated with a catalog of theoretically predicted waveforms.
- Therefore, chances of detecting a generic astrophysical signal depend on the size, scope, and accuracy of the theoretical signal template bank.
- The generation of such a template bank requires many models of the GW emitted from compact binary systems.

Binary black holes

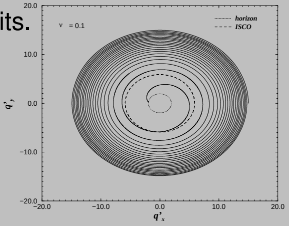
- Optical, radio, and x-ray astronomy have provided us with abundant evidence that many galaxies contain SMBHs in their central nuclei.
- The loudest astrophysical signals in terms of SNR.
- Known examples among galactic binaries.
  - Supermassive – $10^6 - 10^9 M_\odot$.
  - Low frequency sources – space-based detector (LISA)
- Formation processes for stellar mass binaries:
  - Collapse within a binary neutron star system.
  - Capture within a dense region, eg. globular cluster.

Binary black holes



“Chirp” signal from a binary inspiral

- Black holes captured $\rightarrow$ highly elliptical orbits.
- Radiation of gravitational energy
  $\rightarrow$ circularisation of orbits. $\rightarrow$ *inspiral* (PN)
- Decay of orbit leading to
  $\rightarrow$*plunge* (NR) $\rightarrow$ *merger* (NR)
- Single perturbed BH remnant
  $\rightarrow$ exponential *ringdown* to axisymmetric (Kerr) BH.



Einstein equations in 3+1 form

The Einstein equations are manifestly covariant

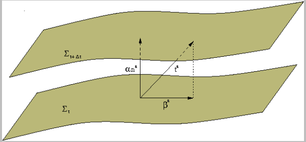
- Need to reformulate as a *Cauchy problem*
- We have ten equations and ten independent components of the four metric $g_{\mu\nu}$, the same number of equations as unknowns.
- Only six of these ten equations involve second time-derivatives of the metric.
- The other four equations, thus, are not evolutions equations. We call these our *constraint equations*.

There are a number of non-unique aspects of the 3+1 decomposition

- Choice of evolution variables
- Choice of gauge
- Binary black hole codes currently use either a harmonic formulation, or a modified (“conformal traceless” or “BSSN”) ADM system.

Numerical Relativity

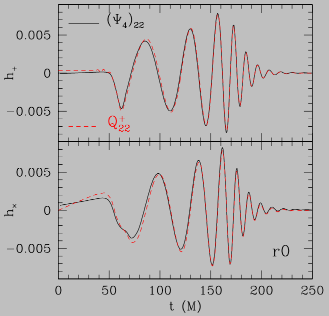
$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi T_{\alpha\beta}$$



- The Einstein equations are a hyperbolic set of nonlinear wave equations for the geometry
- As such, they are most conveniently solved as an initial-boundary-value problem:
  - Assume the geometry is known at some initial time t_0.
  - Evolve the data using the Einstein equations.
  - Prescribe consistent boundary conditions at some finite radius r_0.
- Geometry specified on an initial data slice:
  - metric g_{ab} specifies the intrinsic geometry of the slice.
  - extrinsic curvature determines the embedding in 4D space.
- Evolution equations are integrated using standard numerical methods, eg. Runge-Kutta.
- The equations are differentiated in space on a discrete computational grid using finite differencing methods

Wave extraction

- It has become standard to measure waves as expansions of the Newman-Penrose Ψ_4 scalar.
- An independent method measures gage-invariant perturbations of a Schwarzschild black hole.
- 'Observers' are placed on a 2-sphere at some large radius.
- Measure odd-parity ($Q^{\times}_{lm}$) and even-parity (Q^{+}_{lm}) perturbations of the background metric.



$$h_{+} - \mathrm{i} h_{\times} = \frac{1}{\sqrt{2} r} \sum_{\ell=2}^{\infty} \sum_{m=0}^{\ell} \left(Q^{+}_{\ell m} - \mathrm{i} \int_{-\infty}^{t} Q^{\times}_{\ell m}(t') dt' \right) {}_{-2}Y^{\ell m} ,$$

Newman Penrose

- The Ψ's are defined as components of the Weyl tensor C_{abcd} which is identical to the Riemann tensor R_{abcd}. The complex Weyl scalars Ψ is given by

$$
\begin{aligned}
\Psi_4 &= C_{ab}\,\bar{m}^a\,\bar{m}^b \\
C_{ab} &= R_{ab} - K\,K_{ab} + K_a^{\ c}\,K_{cb} - i\epsilon_a^{\ cd}\,\nabla_d\,K_{bc}
\end{aligned}
$$

- In the chosen a tetrad that separates the radiation part of the Weyl tensor from the non-radiation content. Using spherical coordinates we obtain the tetrad:

$$
\vec{l} \equiv \frac{1}{\sqrt{2}}(\hat{\tau} - \hat{r})\,, \vec{n} \equiv \frac{1}{\sqrt{2}}(\hat{\tau} + \hat{r})\,, \vec{m} \equiv \frac{1}{\sqrt{2}}(\hat{\theta} - i\hat{\phi})\,, \vec{\bar{m}} \equiv \frac{1}{\sqrt{2}}(\hat{\theta} + i\hat{\phi})\,,
$$

- The spacetime metric can be described as

$$
g_{ab} = 2m_{(a}\bar{m}_{b)} - 2n_{(a}l_{b)}
$$

Newman Penrose

- Ψ_4 can be related to the gravitational radiation in the following way: in the transverse-traceless (TT) gauge

$$\frac{1}{4}(\ddot{h}^{TT}_{\hat{\theta}\hat{\theta}} - \ddot{h}^{TT}_{\hat{\phi}\hat{\phi}}) = -R_{\hat{\tau}\hat{\theta}\hat{\tau}\hat{\theta}}\,, \qquad \frac{1}{2}\ddot{h}^{TT}_{\hat{\theta}\hat{\phi}} = -R_{\hat{\tau}\hat{\theta}\hat{\tau}\hat{\phi}} \tag{2}$$

- Finally, we can use $R_{abcd} = C_{abcd}(G_{\mu\nu} = 0)$, to yield the final relation between Ψ_4 and the radiation as a metric perturbation in terms of polarizations

$$\Psi_4 = -(\ddot{h}_+ - i\ddot{h}_\times)\,. \tag{3}$$

- Since two factors of $\vec{\bar{m}}$ each carries a spin-weight of -1, we decompose Ψ_4 in terms of spin-weight -2 spherical harmonics ${}_{-2}Y_{\ell m}(\theta,\phi)$, yielding

$$\Psi_4(t,\vec{r}) = \frac{1}{Mr}\sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} {}_{-2}C_{\ell m}(t)\,{}_{-2}Y_{\ell m}(\theta,\phi)\,, \tag{4}$$

Zerilli Moncrief

- Regge and Wheeler formulated a gauge transformation that allows a radial/angular separation for even and odd parities for some perturbation of GR

$$\begin{aligned}\delta G_{\mu\nu} &= \delta R_{\mu\nu} = \delta\Gamma^{\rho}_{\mu\nu,\rho} - \Gamma^{\rho}_{\mu\rho,\nu}\,, \\ \delta\Gamma^{i}_{jk} &= \frac{1}{2}g^{il}(h_{jl,k} + h_{kl,j} - h_{jk,l})\,,\end{aligned}$$

- Perturbations γ_{ij} can be decomposed using $Y_{\ell m}$ into $\gamma^{\ell m}_{ij}(t,r)$ where

$$\begin{aligned}\gamma_{ij}(t,r,\theta,\phi) &= \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\gamma^{\ell m}_{ij}(t,r) \\ \gamma_{ij}(t,r,\theta,\phi) &= \sum_{k=0}^{6} p_k(t,r)\mathbf{V}_k(\theta,\phi)\end{aligned}$$

  where $\{\mathbf{V}_k\}$ are basis for tensors on a two-sphere.
- In Schwarzschild coordinates, the Regge Wheeler and Zerilli equations may be written

$$\partial_t^2\Psi^{(o/e)}_{\ell m} - \partial_r^2\Psi^{(o/e)}_{\ell m} + V^{(o/e)}_{\ell}\Psi^{(o/e)}_{\ell m} = S^{(o/e)}_{\ell m}$$

  where $S^{(o)}$ is the Regge-Wheeler source function, S are the source function, V are the potential functions.

Zerilli Moncrief

- From $\Psi^{(o)}_{lm}$ and $\Psi^{(e)}_{lm}$ we obtain the gravitational wave amplitude

$$h_+ - ih_\times = \frac{1}{r}\sum_{l,m}\sqrt{\frac{(\ell+2)!}{(\ell-2)!}}(\Psi^{(e)}_{\ell m} + \Psi^{(o)}_{\ell m})_{-2}Y^{\ell m}(\theta,\phi) + \mathcal{O}(\frac{1}{r^2})$$

- The areal coordinate $\hat{r}$ of each extraction sphere is calculated by

$$\hat{r} = \hat{r}(r) = \left[\frac{1}{4\pi}\int\sqrt{\gamma_{\theta\theta}\gamma_{\phi\phi}}d\theta d\phi\right]^{1/2} \longrightarrow S(\hat{r}) = \left(\frac{\partial\hat{r}}{\partial r}\right)^2\int\gamma_{rr}\, d\theta d\phi$$

- We can calculate the six Regge-Wheeler variables, on these spheres by integration of combinations of the metric components over each sphere. From here we can construct the gauge invariant quantities from these Regge-Wheeler and Zerilli variables

$$\begin{aligned} Q^\times_{\ell m} &= \sqrt{\frac{2(l+2)!}{(l-2)!}}\left[c_1^{\times\ell m} + \frac{1}{2}\left(\partial_{\hat{r}}c_2^{\times\ell m} - \frac{2}{\hat{r}}c_2^{\times\ell m}\right)\right]\frac{S}{\hat{r}} \\ Q^+_{\ell m} &= \sqrt{\frac{2(l-1)(l+2)}{l(l+1)}}\frac{(4\hat{r}S^2k_2 + l(l+1)\hat{r}k_1)}{(l-1)(l+2)+6M/\hat{r}}. \end{aligned}$$

This formalism is convenient as it gives us h_+ and $h_\times$ already decomposed into ℓ and m modes with no extra integration stages required.

Horizons

- An event horizon is defined non-locally. Thus it can only be obtained as part of post-processing for any simulation.
- An apparent horizon is defined locally in time on a spacelike slice, and can thus be calculated at each time-step in a simulation.
- It is the outermost smooth closed 2-surface in a slice whose future-pointing outgoing null geodesics have zero expansion, Θ.

$$\Theta \equiv \nabla_i n^i + K_{ij} n^i n^j - K = 0$$

- This outermost surface is coincident with the boundary of a “trapped surface” – i.e. a surface whose future-pointing outgoing null geodesics have *negative* expansion.
- The existence of such a surface automatically implies the existence of a black hole (given certain technical assumptions are met, including energy conditions and a reasonable gauge).

BH Data

- Horizon spins, relative velocities, and masses extracted from the shape and coordinate motion of those black hole horizons.
- The angular momentum of the horizon comes from the horizon ϕ Killing vector field

$$\mathcal{L}_\phi q_{ij} = 0\,,$$

where $q_{ij} := \gamma_{ij} - s_i s_j$ is the induced metric on the horizon for outward-pointing normals s_i. We can then derive the magnitude of the angular momentum from

$$J_H = \frac{1}{8\pi} \oint_S \phi^\ell s^m K_{\ell m} dA\,.$$

- From the angular momentum and horizon surface we can then obtain the mass of the black hole from the equation

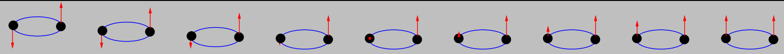
$$M_H^2 = \frac{A_H}{16\pi} + \frac{4\pi J_H^2}{A_H}\,,$$

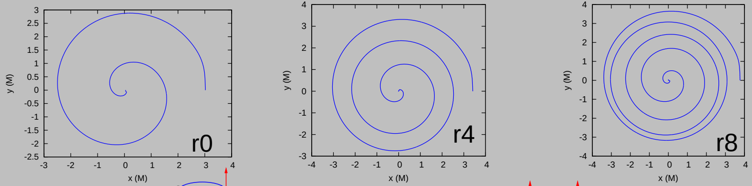
where A_H is the horizon area.

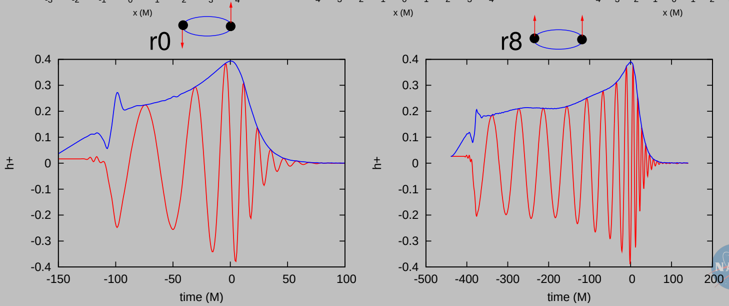
Parameter studies with spinning black holes

Aligned spin leads to an orbital hangup.

r0	r1	r2	r3	r4	r5	r6	r7	r8

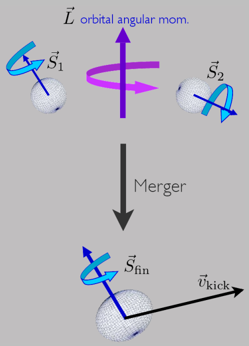






Kicks

- For an equal-mass, non-spinning binary merger, the remnant will be a stationary, spinning black hole.
- If an asymmetry in the bodies is present, the emitted in gravitational waves will also have asymmetry.
- As a result, the remnant black hole will have momentum relative to distant stationary observers, called a recoil or kick.
- Asymmetries in the emitted gravitational wave energy are a result of:
  - Unequal masses.
  - Unequal spin magnitudes.
  - Spins which are misaligned with each other or the orbital angular momentum.



Black hole kicks

- The recoil results from couplings of various wave modes, which are integrated over the entire inspiral time.

$$\mathcal{F}_i \equiv \dot{P}_i = \frac{r^2}{16\pi} \int d\Omega\, n_i \left(\dot{h}_+^2 + \dot{h}_\times^2\right)$$

- PN (2.5) suggests a linear increase of recoil with spin ratio:

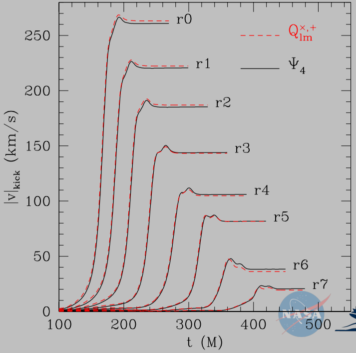
$$|v|_{\rm kick} = c_1 \frac{q^2(1-q)}{(1+q)^5} + c_2 \frac{a_2 q^2(1-qa_1/a_2)}{(1+q)^5} = \tilde{c}_2 a_2 \left(1 - \frac{a_1}{a_2}\right)$$

- In fact, the numerical data points to a quadratic dependence:

$$|v|_{\rm kick} = a_2(c_1 - c_2(\frac{a_1}{a_2}) + c_3(\frac{a_1}{a_2})^2)$$

- The maximum recoil for the anti-aligned case:

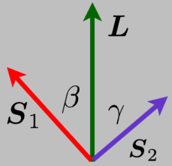
$$|v|_{\rm kick} = 448 \pm 5 km/s$$



Spins

- Equation to predict final spin of merged black hole

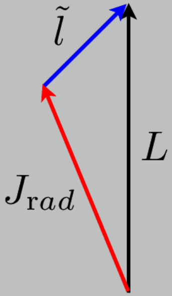
$$|a_{\rm fin}| = \frac{1}{(1+q)^2}\Big[|a_1|^2 + |a_2|^2 q^4 + 2|a_2||a_1|q^2\cos\alpha + 2\left(|a_1|\cos\beta + |a_2|q^2\cos\gamma\right)|\ell|q + |\ell|^2 q^2\Big]^{1/2},$$



where $\cos\alpha \equiv \hat{a}_1\cdot\hat{a}_2$, $\cos\beta \equiv \hat{a}_1\cdot\hat{\ell}$, $\cos\gamma \equiv \hat{a}_2\cdot\hat{\ell}$.

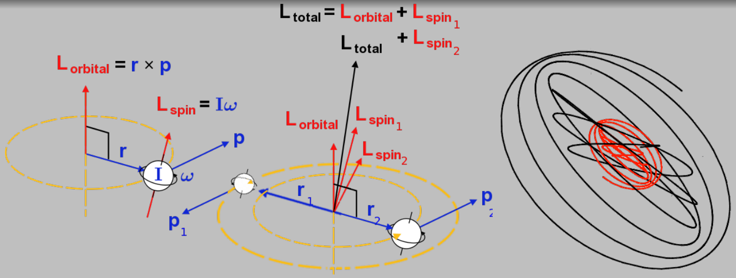
- In order to obtain $|\ell|$ we need to match this equation against general second order polynomial expansions for:
  - Equal mass, unequal but aligned spin binaries
  - Unequal mass, equal spin binaries

$$|\ell| = \frac{s_4}{(1+q^2)^2}\left(|a_1|^2 + |a_2|^2 q^4 + 2|a_1||a_2|q^2\cos\alpha\right) + \left(\frac{s_5\nu + t_0 + 2}{1+q^2}\right)\left(|a_1|\cos\beta + |a_2|q^2\cos\gamma\right) + 2\sqrt{3} + t_2\nu + t_3\nu^2.$$



  - Numerical simulations to obtain s_4, s_5, t_0, t_2, t_3.
  - Test against generic misaligned spin binaries.

Precession in General Relativity



- Spins are not aligned with orbital angular momentum will precess

$$L = L_{orb} + S_1 + S_2\,, \qquad \frac{\partial L}{\partial t} \approx \frac{\partial J_{rad}}{\partial t}$$

  causing whole system to precess, resulting in complex dynamics
- Even without GR binary precession incredible complicated. For EMRI limit for leading order (each term at least 1 page):

$$H = Mc^2 + H_0 + V_1 + V_1 + V_{S1} + V_{S2} + V_{S1,S2} + V_{Q1} + V_{Q1}$$

- the EMRI approximations derived from 1978 plus conservations assumptions currently the only way spins are accounted for in PN

Motivations

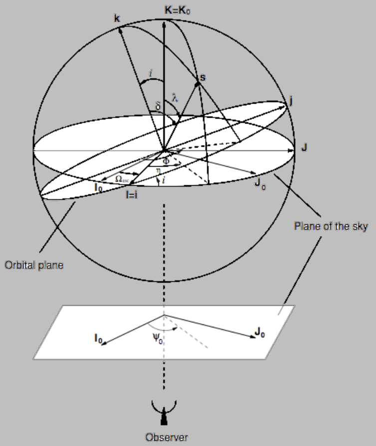
- Parameter studies with simple precession tracking will give us a phenomenological precession model
  - could give us better formulae for SS and SO interaction for EOB and PN
  - combining this model with Wigner rotation in the opposite direction gives us a tool to create generically precession waveforms ...
- Phenomenological templates for precessing systems
  - Cut down the parameter space by 4 degrees of freedom
  - Challenges of NR:
    - Parameter space is large and hard to model generically
    - Simulations are computationally expensive and slow
    - Length and accuracy requirements are not well known
    - Setting up precessing simulations is especially challenging
- Numerically, this is also a tool to get gauge invariant measurements (as opposed to on the horizon)
  - especially for waveforms extracted at null infinity (i.e. CCM/CCE)

Waveform Data

Parameter	Horizon	Waveform
Masses	$M_H^2 = \frac{A_H}{16\pi} + \frac{4\pi J_H^2}{A_H}$	$A(\Psi_4)$
$\hat{L}_{orb}$	$(\Theta \equiv \nabla_i n^i + K_{ij} n^i n^j - K = 0)_z$	$\theta(max(\Psi_{22} - \Psi_{20}))$
$\|L\|$	??	$L_0 - \int \|\hat{L} \cdot \dot{J}\| dt$
Phase/ $\hat{r}$-axis	$(\Theta = 0)_1 - (\Theta = 0)_2$	$\phi(min(\Psi_{20}))$
Separation	$\|(\Theta = 0)_1 - (\Theta = 0)_2\|$	$\sim R_0 - f(J_{Rad}, \dot{\phi})$
S_{fin}	$J_H = \frac{1}{8\pi} \oint_S \phi^\ell s^m K_{\ell m} dA$	Quasinormal
S	$\sum J_{H,i} = \frac{1}{8\pi} \oint_S \phi^\ell s^m K_{\ell m} dA$	$\vec{L} - \vec{J}$
Kick	??	$\frac{r^2}{16\pi} \int d\Omega \, n_i \left(\dot{h}_+^2 + \dot{h}_\times^2\right)$
S_1 and S_2	$J_H = \frac{1}{8\pi} \oint_S \phi^\ell s^m K_{\ell m} dA$	??

Wigner Rotation

- In precessing systems the dominant $\ell = 2, m = \pm 2$ quadrupole mode mixes into non-symmetric modes as the obsever rotates off the L_{orb} axis
- This ruins existing phenomenological waveform formulae
- Rotate reference frame to return to quadrupole dominant system
- Maximize $\Psi_{4,22}$ and minimize $\Psi_{4,20}$
- Tracks the direction of L_{orb} relative to observes and precession of the system



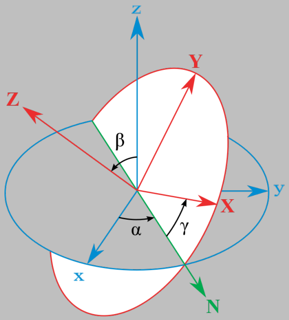
Implementation

- We consider SWSHs of spin weight $s = -2$:

$$\Psi_4^0 = e^{i\omega u} \sum_{\ell m} {}_{-2}\psi_{\ell m}\, {}_{-2}Y_{\ell m}(\theta, \phi),$$

- The transformation of the Weyl scalar $\bar{\Psi}_4^0$

$$\bar{\Psi}_4^0(\theta', \phi') = e^{2i\chi(\theta(\theta'),\phi(\phi'))}\Psi_4^0(\theta(\theta'), \phi(\phi')).$$



Given the transformation of the harmonics

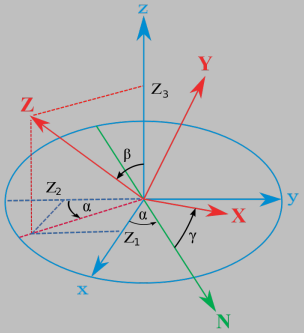
$${}_2Y_{\ell m'}(\theta'(\theta,\phi), \phi'(\theta,\phi)) = \sum_{m=-\ell}^{\ell} D^{(\ell)-1}_{m;m'}\, {}_2Y_{\ell m}(\theta,\phi)\,, \qquad \psi'_{\ell m'} = \sum_{m=-\ell}^{\ell} {}_2\psi_{\ell m} D^{(\ell)-1}_{m;m'}\,,$$

where $D^{(\ell)-1}_{m;m'}$ is the Wigner rotation matrix given by

$$\begin{aligned} D^{(\ell)}_{m;m'}(\theta,\phi) &= e^{-im\phi}\sqrt{(\ell+m)!(\ell-m)!(\ell+m')!(\ell-m')!} \\ &\times \sum_k \frac{-1^{k+m'-m}}{k!(\ell+m-k)!(\ell-m'-k)!(m'-m+k)!} \times \left(\sin\frac{\theta}{2}\right)^{2k+m'-m} \left(\cos\frac{\theta}{2}\right)^{2\ell-2k-m'+m} \end{aligned}$$

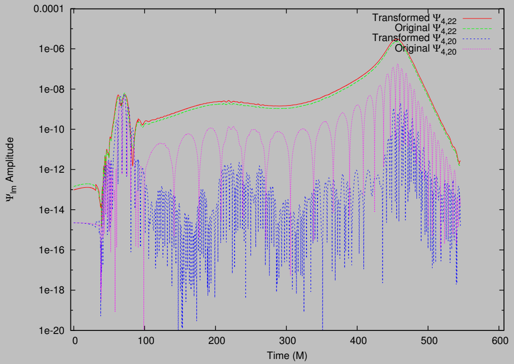
Wigner Rotation

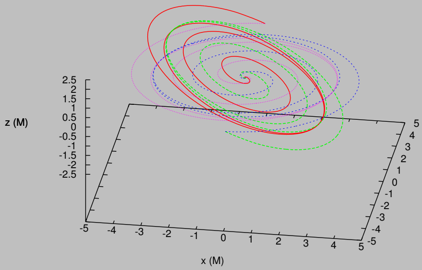
- Use iterative numerical solver to find values of θ, and ϕ for each time that minimize the amplitude of the asymmetric mode ψ'_{20} and maximize ψ'_{22}
- Find rotation angles that place the observer back on the axis of the orbital angular momentum of the system.
- The modes $|m| = 1$ will vanish only when the two black holes can be exchanged by symmetry
- The $m = 0$ optimally is reduced to a non-oscillating mode related to memory effects. This complicates minimization.



Maximizing only the $\ell = 2$, $m = 2$ modes for equal mass systems is perfectly sound. Unequal mass systems have a natural asymmetry that will show in the $\ell = 2$, $m = 1$ modes .

Results for Tilted Simulation





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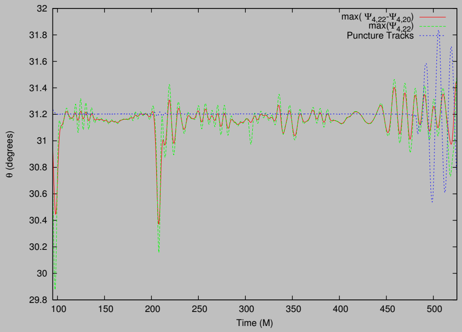
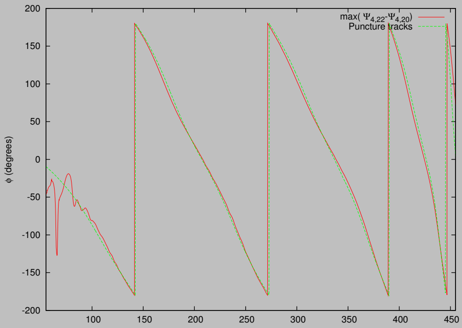
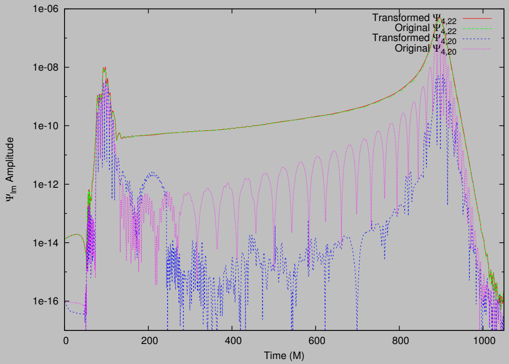
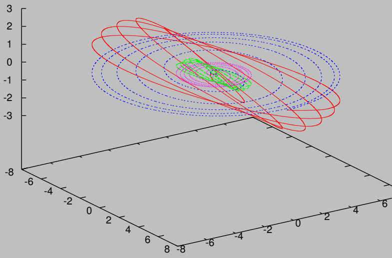


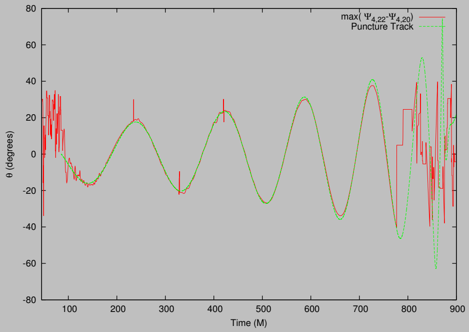


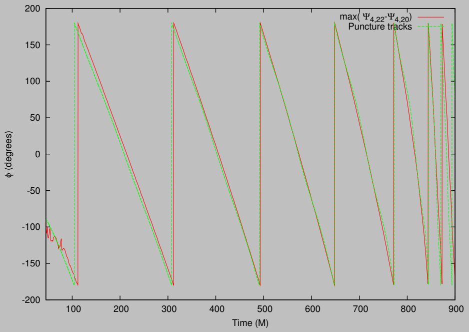
Results For Precessing Simulations





Results For Precessing Simulations





Conclusions

- Accurate method of tracking orbital precession evolution from only observed/extracted waveforms.
- Gauge invariant measures of angular momentum, and spin direction.
- Precession reconstruction from only waveforms data.
- Significant simplification of the emitted GW signal.
- Deprecessed WFs have high overlap with non-precessing equivalents.
- Potential for method for the generation of generic precessing binaries.

Future Work

- Hybrid model of precession rates
  - Wigner rotation of non-precessing runs to cover parameter space
  - Better understanding of SS, SO contributions
- Test of momentum measures accuracy
- Better understanding of mode contributions, for better phenomenological waveform predictions
- Final spin direction predictions

Thank You.