Gravitational-wave Detectability of Equal-Mass Black-hole Binaries With Aligned Spins

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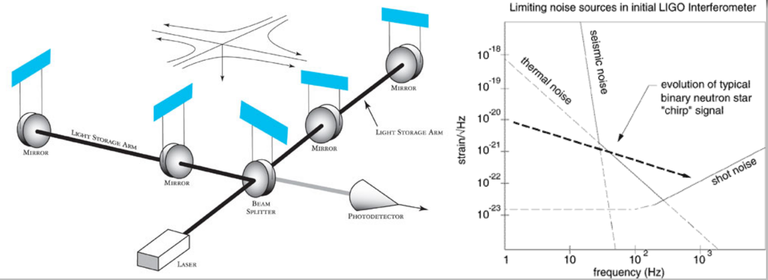
Outline

- Gravitational waves and their detection
- Binary black holes
- Numerical relativity
- Parameter space of aligned spin binaries.
- Construction of waveforms by matching to PN.
- Influence of the spin on detector signal-to-noise.
- The effect of including higher modes.
- Matches between spinning models.

Gravitational waves



- The coupling between matter and geometry is very weak.

$$R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = k T_{\alpha\beta}$$

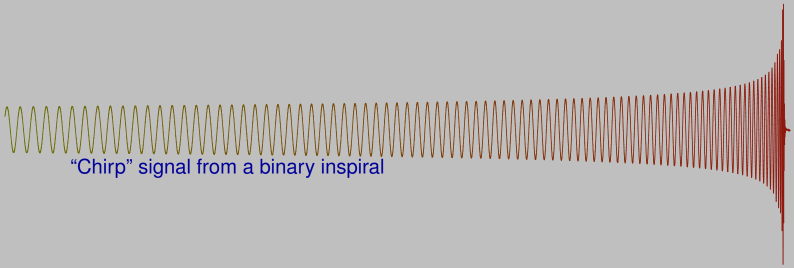
$$k = \frac{8\pi G}{c^4} \simeq 2 \times 10^{-43} \frac{s^2}{m \cdot kg}$$

- Gravitational waves are small features, difficult to detect.
- Unobstructed by intervening matter
- Excellent probe into regions opaque to EM radiation.

Gravitational wave detection

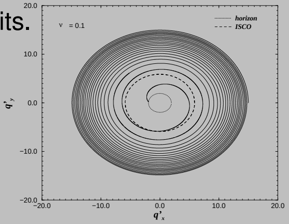
- Currently there are many ground based detectors online which are designed to detect such passing gravitational waves (LIGO, VIRGO, TAMA, GEO).
- Even for binary black hole inspiral and merger, the signal strength is likely to be much less than the level of any detector noise.
- A technique used for this purpose is *matched filtering*, in which the detector output is cross-correlated with a catalog of theoretically predicted waveforms.
- Therefore, chances of detecting a generic astrophysical signal depend on the size, scope, and accuracy of the theoretical signal template bank.
- The generation of such a template bank requires many models of the GW emitted from compact binary systems.

Binary black holes



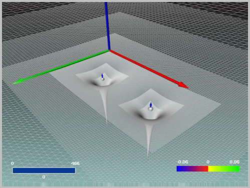
“Chirp” signal from a binary inspiral

- Black holes captured → highly elliptical orbits.
- Radiation of gravitational energy
  → circularisation of orbits. → *inspiral* (PN)
- Decay of orbit leading to
  →*plunge* (NR) → *merger* (NR)
- Single perturbed BH remnant
  → exponential *ringdown* to axisymmetric (Kerr) BH.



Numerical Relativity

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi T_{\alpha\beta}$$

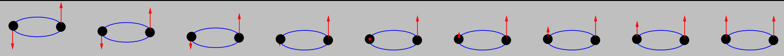


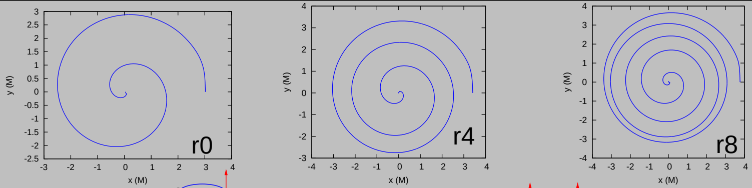
- The Einstein equations are a hyperbolic set of nonlinear wave equations for the geometry
- As such, they are most conveniently solved as an initial-boundary-value problem:
    - Assume the geometry is known at some initial time t_0.
    - Evolve the data using the Einstein equations.
- Geometry specified on an initial data slice:
    - metric g_{ab} specifies the intrinsic geometry of the slice.
    - extrinsic curvature determines the embedding in 4D space.
- Evolution equations are integrated using standard numerical methods, eg. Runge-Kutta.
- The equations are differentiated in space on a discrete computational grid using finite differencing methods

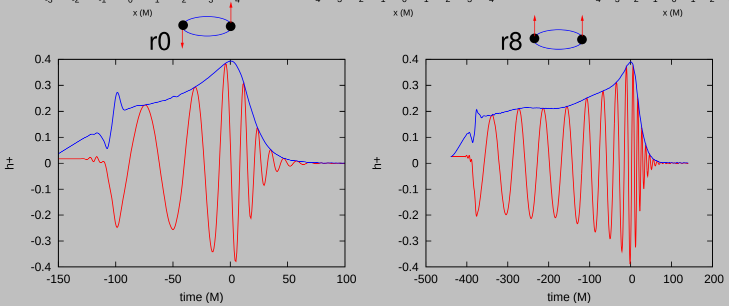
Parameter studies with spinning black holes

Aligned spin leads to an orbital hangup.

r0	r1	r2	r3	r4	r5	r6	r7	r8

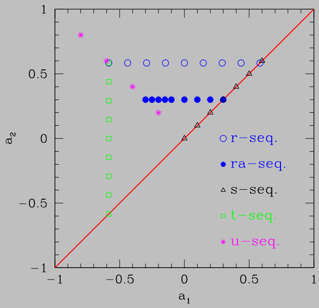


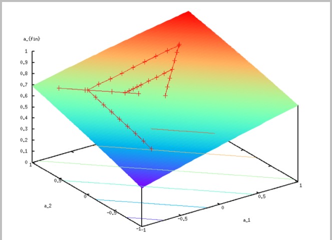




Aligned spin binaries

- We have carried out studies in the parameter space of equal-mass aligned spin binaries, starting from non-eccentric orbit.
- Vary the spin of each BH from $a = -0.6$ to $a = +0.6$.
- Initial studies determined final BH parameters (final spin, radiated energy, kick) as a function of binary parameters.
- Kick depends quadratically on the spin difference, up to $\sim 450 km/s$ in the maximal case.
- Final spin is an almost linear function of the initial spins.

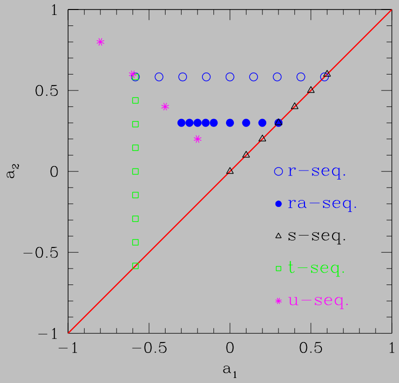




Spin of the final BH.

Aligned spin binaries

- Working towards understanding the properties of waveforms from spinning binaries wrt. data analysis:

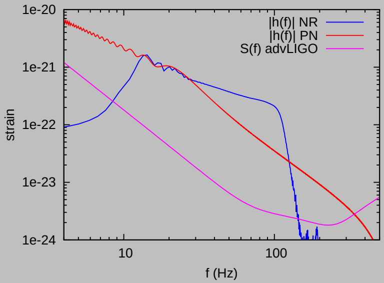


- How is the SNR influenced by the black hole spin?
- Construction of templates: How close are matches between different physical models?
- Which is the “loudest” and which is the “quietest”?
- What is the difference in detector SNR for those waveforms?
- How does it depend on detector, mass of the binary, and number of harmonics?

Matching to PN

- Waveforms determined using Zerilli extraction at radii 100M - 200M.
- Waveforms have 6-11 cycles before merger, $\omega_{\text{ini}} \simeq 0.085/M$ for the "hang-up" case.
- To extend their length into lower freqs., the NR waveforms are matched to PN inspirals.
- Matched in freq. domain against TaylorT4 approximant, non-spinning contributions to 3.5PN, spin to 2.5PN.

- Match performed without adjustment at $\omega_{\text{glue}} = 0.16/M$ for all models.



Signal to noise ratio

- The SNR for a signal h is given by:

$$\rho^2 = \left(\frac{S}{N}\right)^2_{\text{matched}} = 4\int_0^\infty \frac{|\tilde{h}(f)|^2}{S_h(f)}\,df$$

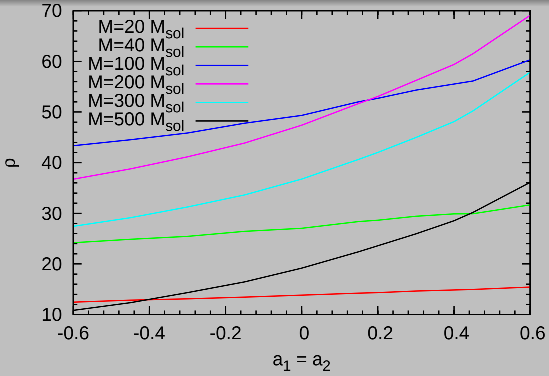
  $S_h(f)$ is the noise power spectral density for a given detector.

- The average SNR over all directions can be calculated as a sum over modes:

$$\begin{aligned}\langle\rho^2\rangle &= \frac{1}{\pi}\int d\Omega \int df \frac{\left|\sum_{\ell m}\tilde{h}_{\ell m}(f)\,{}_{-2}Y_{\ell m}(\Omega)\right|^2}{S_h(f)} \\ &= \frac{1}{\pi}\sum_{\ell m}\int df \frac{|\tilde{h}_{\ell m}(f)|^2}{S_h(f)}.\end{aligned}$$

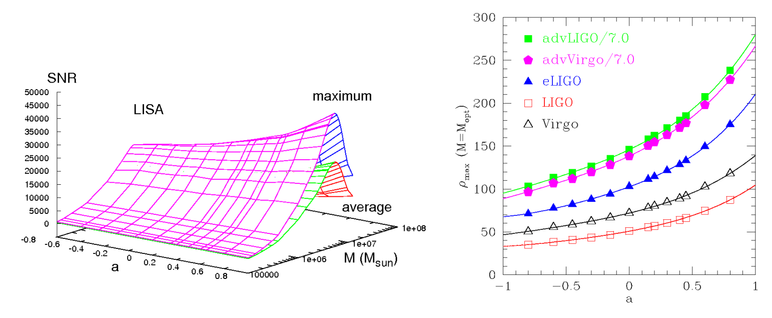
- Comparing the maximum to the average SNR gives an idea of the inhomogeneity of the signal.
- By restricting the sum, we can determine the influence of different mode contributions.
- For the simulations performed, modes up to $l = 4$ are sufficiently resolved.

SNR as a function of spin



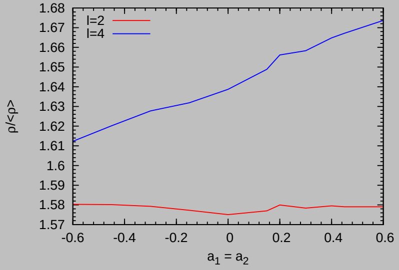
- Equal-spin binaries with maximum spin aligned are more than "three times as loud" as the corresponding binaries with anti-aligned spins, thus corresponding to event rates up to 27 times larger.
- LIGO SNR, $d = 100Mpc$, for various binary masses.
- Approximately quadradic dependence on a/M.
- This is more pronounced at larger masses, where the merger waveform dominates.

Detection

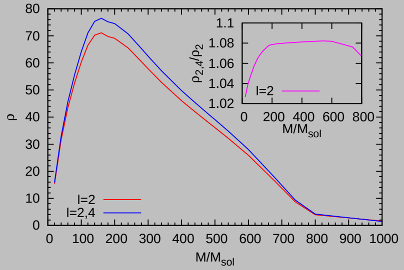


- For any value of a, the maximum horizon distance/SNR also marks the "optimal mass" for the binary $M_{\rm opt}$.
- For any mass, the SNR can be described with a low-order polynomial of the initial spins $\rho = \rho(a_1, a_2)$ and generally it increases with $a \equiv \frac{1}{2}(\boldsymbol{a}_1 + \boldsymbol{a}_2) \cdot \hat{\boldsymbol{L}}$.

SNR contribution by higher modes



Ratio of max/avg SNR for l=2 and l=4 modes ($M = 100M_\odot$).



SNR improvement from including $l = 4$ for model s6 (hang-up).

- The higher mode contributions become more pronounced as the spins are increased.
- For the s6 case, $l = 4$ contribution is from 2.5–8% for larger masses.

Matches between numerical templates

- The match between two waveforms can be computed by the via the weighted scalar product:

$$\langle h_1 | h_2 \rangle = 4\Re \int_0^\infty df \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_h(f)}$$

- The overlap is defined by the normalised scalar product:

$$\mathcal{O}[h_1, h_2] = \frac{\langle h_1 | h_2 \rangle}{\sqrt{\langle h_1 | h_1 \rangle \langle h_2 | h_2 \rangle}}.$$

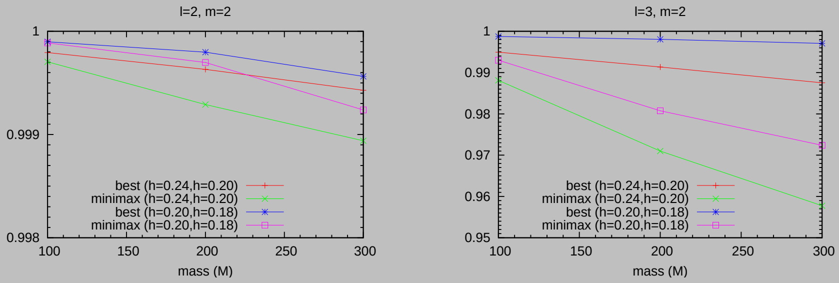
- The *best* match is defined to maximise over the phase of each waveform:

$$M_{\mathrm{best}} \equiv \max_{t_0} \max_{\Phi_1} \max_{\Phi_2} \mathcal{O}[h_1, h_2]$$

- The *minimax* maximises over the phase of one, and minimises over the other (worst case):

$$M_{\mathrm{minimax}} \equiv \max_{t_0} \min_{\Phi_2} \max_{\Phi_1} \mathcal{O}[h_1, h_2]$$

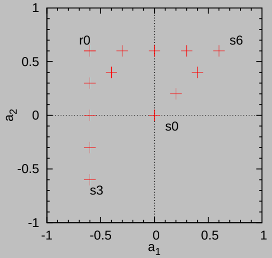
Resolution dependence

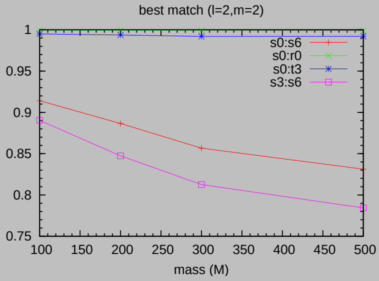


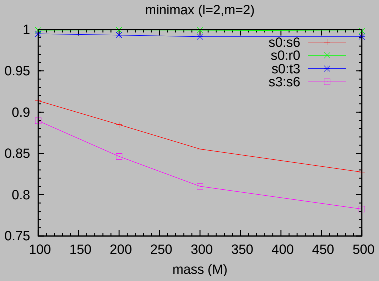
- Influence of numerics can be gauged by computing the match of waves computed at different resolutions.
- Consider only the numerically generated (not PN extended) waveforms for large masses.

Matches between models

- Very good (>0.998) matches along the NW-SE diagonal.
- Poorer match along NE-SW, degrading with increasing total spin.
- Reflects results concerning "universality" of the plunge.







Summary

- SNR increases approximately quadratically with total spin. Spin-aligned models have approx. double the SNR of anti-aligned models.
- Higher mode contribution to SNR increases with total spin.
- The dominant $l = 2, m = 2$ mode is very similar for models with the same total spin. The match degrades as the total spin is increased.
- The waveform from a nonspinning binary can be extremely useful across the *whole* spin diagram and yield very large overlaps even for binaries with very high spins.
- This result is reassuring in light of the fact that most of the searches in the detector data are made using phenomenological waveforms based on nonspinning binaries.
- The diagonal $a_1 = -a_2$ (the u sequence) cannot be distinguished within our given numerical accuracy, whereas configurations along the diagonal $a_1 = a_2$ (the s sequence) are clearly different.

Thank You.

Publications

- L. Rezzolla, P. Diener, E. N. Dorband, D. Pollney, C. Reisswig, E. Schnetter, JS. **The Final Spin From the Coalescence of Aligned-spin Black-hole Binaries**. *Astrophys. J.* **674** *(2008) L29*. Preprint: arXiv.org:0710.3345 [gr-qc]
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- P. Ajith, *et al.* (including JS). **“Complete” gravitational-waveforms for black-hole binaries with non-precessing spins**. Preprint: arXiv:0909.2867 [gr-qc]