Final Spin from Binary Black Hole Coalescence

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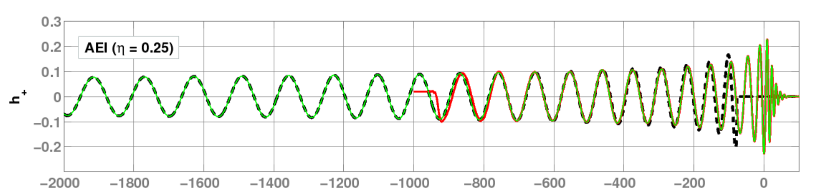
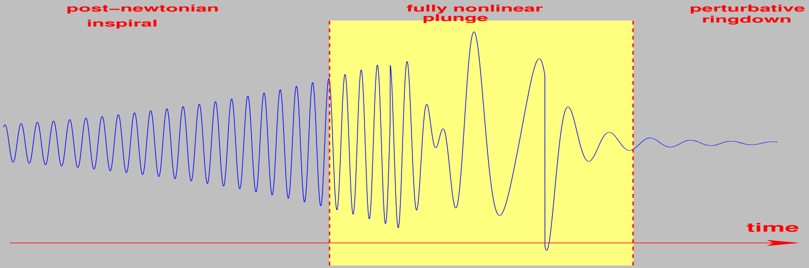
Outline

The binary black hole problem

- Radiation-reaction (the emission of gravitational waves) causes the orbits to shrink.
- At large separation, eccentricity decays at a faster rate than the orbit decay:
  - The late inspiral is expected to be quasi-circular for astrophysical models.
- After merger, a single distorted black hole is formed
  - Perturbations decay exponentially followed by a power-law tail
  - Quasi-normal mode ringing.
- Result: A waveform with gradually increasing frequency and amplitude, followed by a sharp cutoff after merger:



The binary black hole problem



- Qualitative features of the merger "chirp" waveform are known.
- Different approximations are appropriate in different regimes:
  - Post-Newtonian at large separations.
  - Numerical simulation for the last orbits and merger.
  - Perturbative techniques for the ringdown.

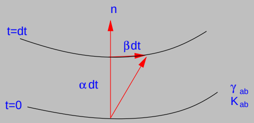
Numerical relativity

- We are interested to accurately determine the gravitational wave content and physical properties of spacetimes which are:
  - Strong.
  - Dynamical.
  - Without symmetries.
- In the strong-field, dynamical regime, nonlinear terms of the Einstein equation play an important role – approximations break down.
- We resort to numerical computation (computer simulation) to determine solutions of the Einstein equations:

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi T_{\alpha\beta}$$

- For the purpose of this talk, we consider only vacuum solutions, ie. $T_{\alpha\beta} = 0$.

3+1 decomposition of Einstein equations



- Foliate $\mathcal{M}$ with a set of spacelike 3-D hypersurfaces Σ_t, parametrised by t.
- Decompose the trajectories of t into components normal and parallel to Σ_t

$$t^\mu = \alpha n^\mu + \beta^\mu$$

  - α is called the "lapse", and fixes the distance between successive slices.
  - β^μ is the "shift", and defines how coordinates move within the slice.
  - These quantities are entirely gauge, ie. can be freely chosen, do not influence the physics.
  - 3+1 line element:

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt).$$

3+1 decomposition of Einstein equations

- The choice of normal n^α naturally induces a metric on each slice via:

$$\gamma_{\alpha\beta} = g_{\alpha\beta} + n_\alpha n_\beta$$

- The mixed form of $\gamma_{\alpha\beta}$ projects tensors onto the spacelike hypersurfaces:

$$\perp^\alpha{}_\beta = \delta^\alpha{}_\beta + n^\alpha n_\beta$$

- Associated compatible covariant derivative in slices

$$D_\alpha := \perp^\mu{}_\alpha \nabla_\mu,$$

$$D_\alpha \gamma_{\beta\gamma} = 0.$$

- The extrinsic curvature (describing the embedding of Σ in $\mathcal{M}$) is given by:

$$K_{\alpha\beta} = -\perp_\alpha{}^\mu \perp_\beta{}^\nu \nabla_{(\mu} n_{\nu)} = -\frac{1}{2}\mathcal{L}_{\mathbf{n}}\gamma_{\alpha\beta}$$

3+1 decomposition of Einstein equations

- The 4D Einstein equations can be written out explicitly in terms of derivatives of the spatial metric and the extrinsic curvature.
  Evolution equations (6+6):

$$
\begin{aligned}
(\partial_t - \mathcal{L}_\beta)\gamma_{ab} &= -2\alpha K_{ab} \\
(\partial_t - \mathcal{L}_\beta)K_{ab} &= -\nabla_a\nabla_b\alpha + \alpha(R_{ab} + KK_{ab} - 2K_{ai}K^i{}_b)
\end{aligned}
$$

  Constraints (1+3):

$$
\begin{aligned}
\mathcal{H} &= R + K^2 - K_{ij}K^{ij} = 0 && \text{(hamiltonian)} \\
\mathcal{M}_a &= \nabla^i(K_{ai} - \gamma_{ai}K) = 0 && \text{(momentum)}
\end{aligned}
$$

- Cauchy problem for the ADM formulation of Einstein's equations:
  - Prescribe $\{\gamma_{ab}, K_{ab}\}$ at $t = 0$ subject to the constraints,
  - Specify coordinates via α and β^a,
  - Evolve data to future using Einstein eqs and definition of K_{ab}.

Reduction to explicitly hyperbolic form

- Expanding

$$R_{\alpha\beta} = 0,$$

  we get a PDE whose principle part contains mixed 2nd-derivatives of the metric:

$$-\frac{1}{2}\Box g_{ab} - \frac{1}{2}g^{ij}\left(g_{ij,ab} - g_{ia,bj} - g_{ib,aj}\right) + g^{ij}\left(\Gamma^{k}{}_{ai}\Gamma_{jkb} - \Gamma^{k}{}_{ab}\Gamma_{ijk}\right) = 0.$$

- Harmonic gauge: Mixed 2nd derivatives can be removed by introducing the new variables $-g^{ai}{}_{,i} = \Gamma^{a}$ (DeDonder 1921, Choquet-Bruhat 1952):

$$\Box g_{ab} = 2g_{i(a}\partial_{b)}\Gamma^{i} + 2\Gamma^{i}\Gamma_{(ab)i} + 2g^{ij}\left(2\Gamma^{k}{}_{i(a}\Gamma_{b)kj} + \Gamma^{k}{}_{ai}\Gamma_{kjb}\right)$$

- The Einstein equations are explicitly in the form of a 2nd order wave equation for the metric.
- Hyperbolicity, and thus stability, follows directly from the reduction to harmonic form.

Evolution equations: "BSSN" Formulation

(Kojima, Nakamura, Oohara 1987, Shibata, Nakamura 1995, Baumgarte, Shapiro 1999)

Key idea: Reformulate ADM by changing variables according to certain geometrical and stability criteria.

1. Conformally decompose the 3-metric:

$$\tilde{\gamma}_{ab} = e^{-4\phi}\gamma_{ab}$$

Introduce the conformal factor as an evolution variable, and subject to the algebraic constraint $\det \tilde{\gamma}_{ab} = 1$

2. Evolve the trace of the extrinsic curvature as a separate variable.

$$\begin{aligned} \phi &= \frac{1}{4}\log\psi & \tilde{\gamma}_{ab} &= e^{-4\phi}\gamma_{ab} \\ K &= \gamma^{ij}K_{ij} & \tilde{A}_{ab} &= e^{-4\phi}(K_{ab} - \frac{1}{3}\gamma_{ab}K) \end{aligned}$$

3. Introduce evolution variables (gauge source functions):

$$\tilde{\Gamma}^a = \tilde{\gamma}^{ij}\tilde{\Gamma}^a_{ij} = -\partial_i\tilde{\gamma}^{ai}$$

BSSN evolution equations

- Evolution equations:

$$
\begin{aligned}
(\partial_t + \mathcal{L}_\beta)\tilde{\gamma}_{ab} &= -2\alpha\tilde{A}_{ab} \\
(\partial_t + \mathcal{L}_\beta)\phi &= -\frac{1}{6}\alpha K\phi \\
(\partial_t + \mathcal{L}_\beta)\tilde{A}_{ab} &= e^{-4\phi}(D_a D_b \alpha + \alpha R_{ab})^{\mathrm{TF}} + \alpha(K\tilde{A}_{ab} - 2\tilde{A}_{ai}\tilde{A}^i{}_b) \\
(\partial_t + \mathcal{L}_\beta)K &= -D^i D_i \alpha + \alpha(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2) \\
\partial_t \tilde{\Gamma}^a &= \tilde{\gamma}^{ij}\partial_i\partial_j\beta^a + \frac{1}{3}\tilde{\gamma}^{ai}\partial_i\partial_j\beta^j + \beta^i\partial_i\tilde{\Gamma}^a - \tilde{\Gamma}^i\partial_i\beta^a + \frac{2}{3}\tilde{\Gamma}^a\partial_i\beta^i \\
&\quad - 2\tilde{A}^{ai}\partial_i\alpha + 2\alpha(\tilde{\Gamma}^a{}_{ij}\tilde{A}^{ij} + 6\tilde{A}^{ai}\partial_i\phi - \frac{2}{3}\tilde{\gamma}^{ai}\partial_i K)
\end{aligned}
$$

- Constraints:

$$
\begin{aligned}
\mathcal{H} &= R + \frac{2}{3}K^2 - \tilde{A}_{ij}\tilde{A}^{ij} = 0 \\
\mathcal{M} &= D_j\left(e^{4\phi}\tilde{A}^{ij} - \frac{2}{3}e^{4\phi}\tilde{\gamma}^{ij}K\right) = 0 \\
\mathcal{G}^a &= \tilde{\Gamma}^a - \partial_i\tilde{\gamma}^{ai} = 0 \\
\mathcal{A} &= \tilde{\gamma}^{ij}\tilde{A}_{ij} = 0 \\
\mathcal{S} &= \det\tilde{\gamma} - 1 = 0
\end{aligned}
$$

Coordinate conditions

There are a number of features we'd like to see in a good choice of coordinates:

- Cover regions of spacetime of interest
- Simplify equations of motion
  - eliminate evolution variables
  - recast equations into nice form (eg. harmonic coords)
- Simplify the physics (eg. reduce dynamics on the numerical grid)
  - minimal distortion (Smarr-York 1978), "symmetry seeking" (Garfinkle-Gundlach 1999)
  - co-rotating coordinates
  - known asymptotic states
- Avoid physical singularities
- Computationally efficient
- Compatible with hyperbolicity, well-posedness

Coordinate conditions

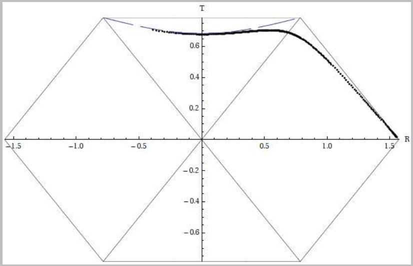
- The Bona-Massó slicing conditions have become generic for BSSN-type evolutions.

$$(\partial_t - \beta^i \partial_i)\alpha = -\alpha^2 f(\alpha) K$$

with $f(\alpha) > 0$.

  - $f = 0$: Geodesic slicing
  - $f \to \infty$: Maximal slicing
  - $f = 1$: Harmonic slicing
  - $f = 2/\alpha$: "$1 + \log$" slicing

- They are evolution equations – inexpensive to compute.
- In particular, the $1 + \log$ variant has excellent and well understood singularity avoiding properties [Hannam et al. 2006].



Initial data for binary black holes

Misner data:
Two isometric, time symmetric, conformally flat, sheets connected by N black holes, solved as infinite series expansion.

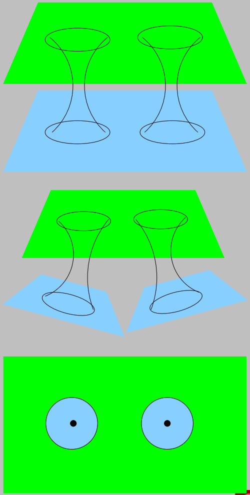
Brill-Lindquist:
Conformally flat, time symmetric, hamiltonian constraint solved by: $\psi = 1 + \Sigma_{i=1}^{N} \frac{m_i}{2r_i}$.

Puncture:
Assume a conformal factor of the form:

$$\psi = u + \Sigma_{i=1}^{N} \frac{m_i}{2r_i}.$$

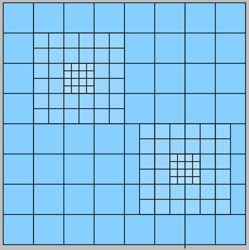
Find C^2 solutions for u of the hamiltonian constraint:

$$\tilde{\nabla}^2 u + \frac{1}{8}\chi^7 \tilde{A}_{ij}\tilde{A}^{ij}(1+\chi u)^{-7}) = 0$$



Discretisation of the Einstein Equations

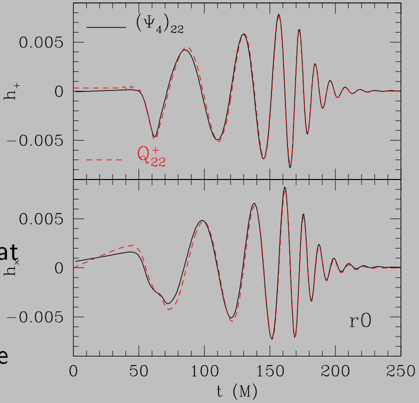
- The evolution of the field variables is carried out using a method of lines technique:
  Time evolution is carried out using standard ODE techniques, such as Runge-Kutta.
- The evolution equations consists of any suitable spatial discretisation of the RHS of the field equations.
  Eg: Finite differences. Spectral methods. Finite elements.
- For the results in this talk we use finite differencing:



- Exact in the limit of infinite resolution.
- Higher order methods converge more quickly with resolution, require larger stencils.
- For efficiency, resolution is concentrated in the strong field regions – "Adaptive Mesh Refinement"

Wave extraction

- It has become standard to measure waves as expansions of the Newman-Penrose Ψ_4 scalar.
- An independent method measures gage-invariant perturbations of a Schwarzschild black hole.
- 'Observers' are placed on a 2-sphere at some large radius.
- Measure odd-parity ($Q^{\times}_{lm}$) and even-parity (Q^{+}_{lm}) perturbations of the background metric.



$$h_+ - i h_\times = \frac{1}{\sqrt{2}r} \sum_{\ell=2}^{\infty} \sum_{m=0}^{\ell} \left(Q^{+}_{\ell m} - i \int_{-\infty}^{t} Q^{\times}_{\ell m}(t') dt' \right) {}_{-2}Y^{\ell m} \, ,$$

Cactus

- BSSN 1st order in space, 2nd order in time
- 1+log lapse, Γ-driver shift evolution

$$(\partial_t - \beta^i \partial_i)\alpha = -2\alpha K$$

$$\partial_t \beta^i = k \partial_t \tilde{\Gamma}^i \qquad (k > 0)$$

- Carpet Adaptive Mesh Refinement follows Puncture movement
- Apparent Horizon Finder and Isolated Horizons
- Wave Extraction both with Zerrilli Extraction and the Newman Penrose Ψ_4
- Puncture initial data with PN derived orbital parameters

BH Astrophysics with Numerical Relativity

- There are a number of interesting physics results available from studying the last orbits, plunge and ringdown.
  - State of the final BH from generic initial data
  - Recoil of the final BH
  - Mode decomposition of the plunge waveform.
- These results are easily accessible, given reasonable quasi-circular/PN orbit parameters at late times.

In a series of papers, we have studied the merger physics of binary-BHs with spins:

- Koppitz et al. "Recoil Velocities from Equal-Mass Binary-Black-Hole Mergers", PRL **99**, 041102 (2007).
- Pollney et al. "Recoil velocities from equal-mass binary black-hole mergers: a systematic investigation of spin-orbit aligned configurations",
- Rezzolla et al., "Spin Diagrams for Equal-Mass Black-Hole Binaries with Aligned Spins",
- Rezzolla et al., "The final spin from the coalescence of aligned-spin black-hole binaries",
- Rezzolla et al., "On the final spin from the coalescence of two black holes",

Kicks from binary black hole mergers

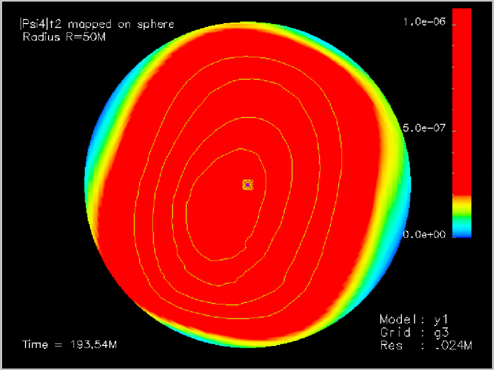
- For an equal-mass, non-spinning binary merger, the remnant will be a stationary, spinning black hole.
- If an asymmetry in the bodies is present, the emitted in gravitational waves will also have asymmetry.
- As a result, the remnant black hole will have momentum relative to distant stationary observers, called a recoil or kick.
- Asymmetries in the emitted gravitational wave energy are a result of:
  - Unequal masses.
  - Unequal spin magnitudes.
  - Spins which are misaligned with each other or the orbital angular momentum.
- The recoil velocity has large implications for simulations of stellar clusters and galaxies.

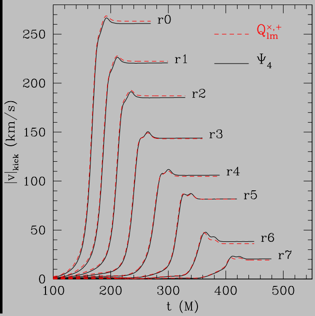
Black hole kicks

- The recoil results from couplings of various wave modes, which are integrated over the entire inspiral time.

$$\mathcal{F}_i \equiv \dot{P}_i = \frac{r^2}{16\pi} \int d\Omega \, n_i \left(\dot{h}_+^2 + \dot{h}_\times^2 \right)$$

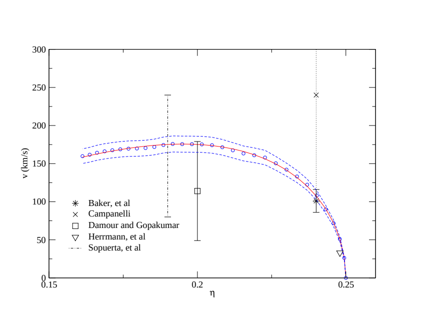
- It is only in the last 1-2 orbits before merger that the recoil becomes significant.



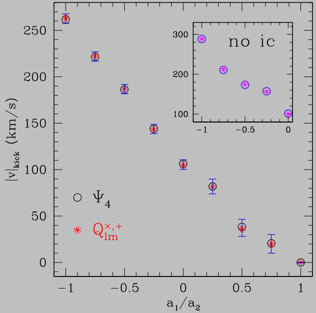


Black hole kicks

- The first numerical calculations for unequal mass systems [Goddard, Penn State, Jena] provided a strong validation of earlier analytical estimate due to [Fitchett 83].
- Soon after, it was determined that much higher kick velocities are obtainable when spins are unequal [Penn State, AEI, UTB].
- In the anti-aligned case, a maximum recoil of: $|v|_{\text{kick}} = 448 \pm 5\text{km/s}$



[Gonzalez et al. 2006]



[Pollney et al. 2007]

Recoil velocities from spinning BHs

- PN (2.5) suggests a linear increase of recoil with spin ratio:

$$|v|_{\rm kick} = c_1 \frac{q^2(1-q)}{(1+q)^5} + c_2 \frac{a_2 q^2 (1 - q a_1/a_2)}{(1+q)^5} = \tilde{c}_2 a_2 \left(1 - \frac{a_1}{a_2}\right)$$

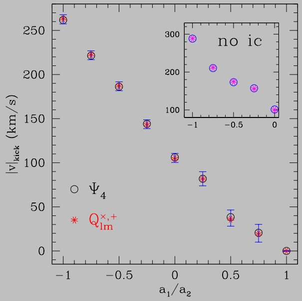
- In fact, the numerical data points to a quadratic dependence:

$$|v|_{\rm kick} = a_2 (c_1 - c_2 (\frac{a_1}{a_2}) + c_3 (\frac{a_1}{a_2})^2)$$

- The maximum recoil for the anti-aligned case:

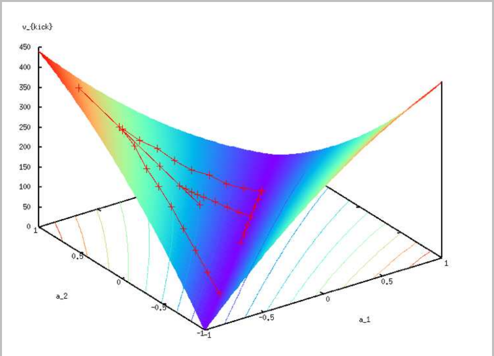
$$|v|_{\rm kick} = 448 \pm 5 km/s$$

- Excellent agreement with other published numrel data.



Recoil velocities

- The recoil velocity of the final BH can be fit to a quadratic function of the initial BH spins (a_1, a_2):



$$|v_{\rm kick}| = |c_1(a_1 - a_2) + c_2(a_1^2 - a_2^2)|\,.$$

$$c_1 = -220.97 \pm 0.78\,, \qquad c_2 = 45.52 \pm 2.99$$

Zero kick when $a_1 = a_2$ Linear scaling along $a_1 = -a_2$

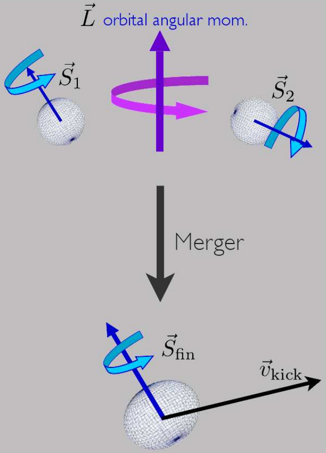
Black Hole Spins

- The objective of this talk is to derive a phenomenological formula for spin of a black hole resulting from the merger of two black holes of arbitrarily oriented spins and generic mass ratio
- This has applications for:
  - statistical distribution of black hole properties
  - simulations of the central regions of galaxies
  - dynamics of star clusters
- We need to simulate 2 spinning black holes over a 7D parameter space

$$\{S_1^i, S_2^i, M_1/M_2\}$$

to get one final black hole

$$\{v_{kick}^i\}\{S_{fin}^i/M_{fin}^2\}$$



- We obtain a general 2nd order polynomial expansion with 5 restricting assumptions for our coefficients:
  - mass radiated in graviational waves may be neglected, $M_{fin} \approx M$:

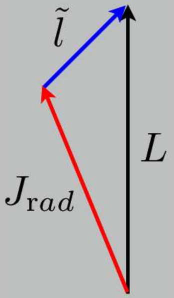
$$M_{rad}/M = 1 - M_{fin}/M \approx 5 - 7x10^{-2}$$

  - magnitude of the final spin vector is the sum of the initial spin vectors flus a thrid vector, $\tilde{\ell}$:

$$S_{fin} = S_1 + S_2 + \tilde{\ell}$$

    the vector ℓ is the difference between the orbital angular momentum when the binary is widely separated L, and the angular momentum radiated up to the merger $\ell = L - J_{\mathrm{rad}}$.
  - The vector $\tilde{\ell}$ is parallel to L (correct by equatorial symmetry for spins aligned with L) with a resulting error in the estimate of $\sim |J^{\perp}_{\mathrm{rad}}|^2/|\tilde{\ell}|^2 \sim |J^{\perp}_{\mathrm{rad}}|^2/(2\sqrt{3}M_1 M_2)^2$ these errors are small in all the configurations that we have analysed
  - When the initial spin vectors are equal and opposite ($S_1 = -S_2$) and the masses are equal ($q = 1$), the spin of the final black hole is the same as for the nonspinning binaries
  - The extreme mass ratio limit (EMRL) is trivial

$$S_{fin} = S_1 \qquad if \qquad M \mapsto 0$$



Black Hole Spins

- Using these assumptions, it follows that:

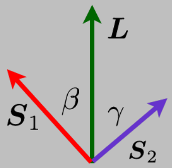
$$|a_{\text{fin}}| = \frac{1}{(1+q)^2}\Big[|a_1|^2 + |a_2|^2 q^4 + 2|a_2||a_1|q^2\cos\alpha +$$
$$2\left(|a_1|\cos\beta + |a_2|q^2\cos\gamma\right)|\ell|q + |\ell|^2 q^2\Big]^{1/2},$$

where $\cos\alpha \equiv \hat{a}_1\cdot\hat{a}_2$, $\cos\beta \equiv \hat{a}_1\cdot\hat{\ell}$, $\cos\gamma \equiv \hat{a}_2\cdot\hat{\ell}$.

- In order to obtain $|\ell|$ we need to match this equation against general second order polynomial expansions for:
  - Equal mass, unequal but aligned spin binaries
  - Unequal mass, equal spin binaries

$$|\ell| = \frac{s_4}{(1+q^2)^2}\left(|a_1|^2 + |a_2|^2 q^4 + 2|a_1||a_2|q^2\cos\alpha\right) +$$
$$\left(\frac{s_5\nu + t_0 + 2}{1+q^2}\right)\left(|a_1|\cos\beta + |a_2|q^2\cos\gamma\right) +$$
$$2\sqrt{3} + t_2\nu + t_3\nu^2 .$$

  - Numerical simulations to obtain s_4, s_5, t_0, t_2, t_3.
  - Test against generic misaligned spin binaries.



Final spin via horizon shape

- Valid once a common horizon has formed and settled down to a perturbed state.
- Measure equatorial circumference C_e and polar circumference C_p along orthogonal great circles.
- $C_r = C_p/C_e$ settles to a constant value:

$$C_r(j) = \frac{1+\sqrt{1-j^2}}{\pi} E\left(-\frac{j^2}{(1+\sqrt{1-j^2})^2}\right)$$

where $j = a/M$, and $E(k)$ is the complete elliptic integral of the second kind

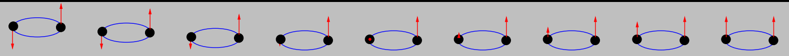
$$E(k) = \int_0^{\pi/2} \sqrt{1-k\sin^2\theta}d\theta.$$

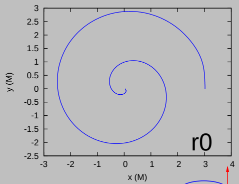
- This equation is integrated numerically to obtain j from the horizon shape.

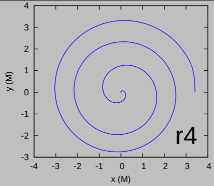
Parameter studies with spinning black holes

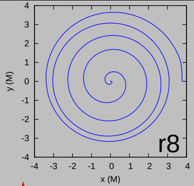
Aligned spin leads to an orbital hangup.

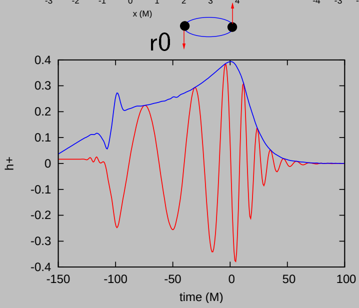
r0	r1	r2	r3	r4	r5	r6	r7	r8

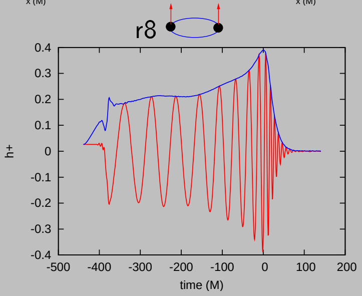






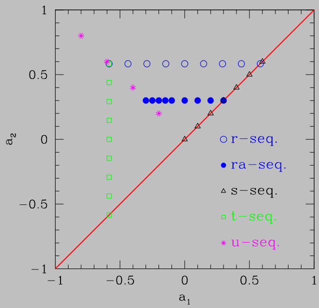


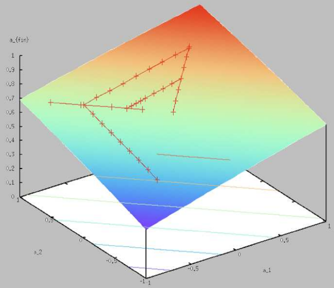




Equal Mass, Aligned Unequal Spin Binaries

- We have carried out studies in the parameter space of equal-mass aligned spin binaries, starting from non-eccentric orbit.
- Vary the spin of each BH from $a = -0.6$ to $a = +0.6$.
- Initial studies determined final BH parameters (final spin, radiated energy, kick) as a function of binary parameters.
- Kick depends quadratically on the spin difference, up to $\sim 450 km/s$ in the maximal case.
- Final spin is an almost linear function of the initial spins.





Spin of the final BH.

Aligned Unequal Spins, Equal Mass

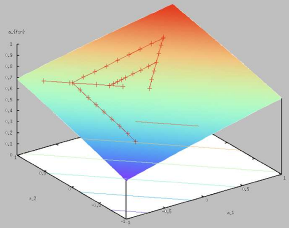
The resulting expression is:

$$a_{\rm fin} = p_0 + p_1(a_1 + a_2) + p_2(a_1 + a_2)^2 \,.$$

with

$$p_0 = 0.6883 \pm 0.0003, \quad p_1 = 0.1530 \pm 0.0004, \quad p_2 = -0.0088 \pm 0.0005,$$

$$p_0 = \frac{\sqrt{3}}{2} + \frac{t_2}{16} + \frac{t_3}{64}, \quad p_1 = \frac{1}{2} + \frac{s_5}{32} + \frac{t_0}{8}, \quad p_2 = \frac{s_4}{16}.$$



- predicts a minimum and maximum spin:

$$(a_{fin})_{min} \approx 0.347$$

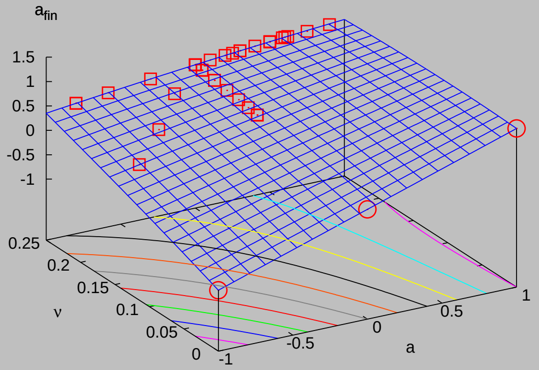
$$(a_{fin})_{max} \approx 0.959$$

for alligned spins.

Unequal Mass, Aligned Spins

- The spin of the final black hole has been determined for very generic initial conditions:
  - Arbitrary aligned spins
  - Unequal masses
- In the extreme-mass-ratio limit, approximation methods can be used.

$$a_{\rm fin} = a + s_4 a^2\nu + s_5 a\nu^2 + t_0 a\nu + 2\sqrt{3}\nu + t_2\nu^2 + t_3\nu^3$$



$$s_4 = -0.129 \pm 0.012$$
$$s_5 = -0.384 \pm 0.261$$
$$t_0 = -2.686 \pm 0.065$$
$$t_2 = -3.454 \pm 0.132$$
$$t_3 = 2.353 \pm 0.548$$

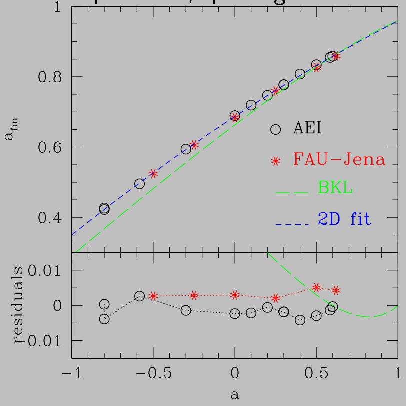
Accuracy for Aligned spins

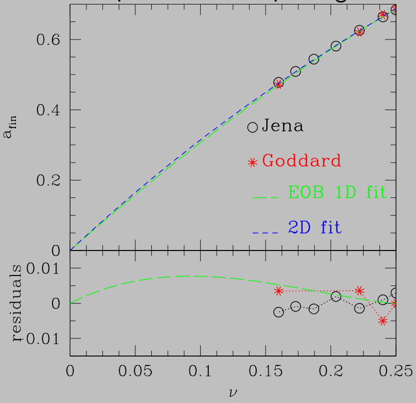
- Numerical relativity results for non-spinning BHs (Jena, Goddard, Penn State)
- Extreme mass ratio calculations for the $m_1 \gg m_2$ limit (Buonanno-Kidder-Lehner 2007)

$$a_{\rm fin} = a + s_4 a^2 \nu + s_5 a \nu^2 + t_0 a \nu + 2\sqrt{3}\nu + t_2 \nu^2 + t_3 \nu^3$$

Equal-mass, spinning

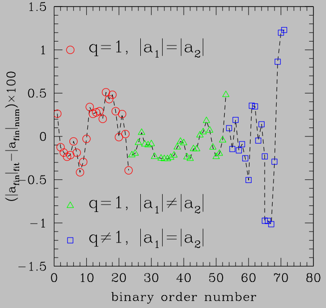
Unequal mass, nonspinning



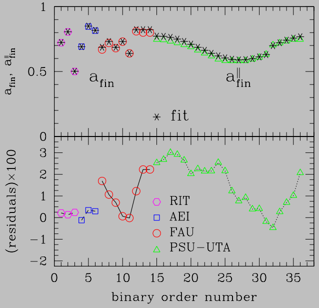


Accuracy for Misaligned Spins

Aligned spinning



Misaligned spinning



- Circles refer to equal-mass, equal-spin binaries in (Rezzolla-etal, Marronetti-etal, Berti-etal, Buonanno-etal 2007),
- Triangles to equal-mass, unequal-spin binaries in (Rezzolla-etal, Berti-etal),
- Squares to unequal-mass, equal-spin binaries in (Berti-etal,Buonanno-etal,Rezzolla-etal).

- Hexagons refer to data from ref. (Campanelli:2006vp)
- Squares to the data obtained in AEI runs
- Circles to data from ref. (Tichy:2007gso)
- Triangles to data from ref. (Herrmann:2007ex)

Summary

- Binary black holes are a fertile ground for gravitational physics (recoil, spin, waveforms).
  - Modellinog of final spins and kicks within ia few percent precision
  - Hybrid methods, combining post-Newtonian and perturbative approaches with numerical results are starting to provide a picture of the full inspiral-merger process.
- Techniques for numerical relativity are now rather advanced. There are still systematic problems to be tackled:
  - Efficiency.
  - Improving initial data construction.
  - Understanding limitations of wave extraction at a finite radius.
- Room for improvement by including J_{rad} in the orbital plane
- Interesting to see what we can discover about extremal spin regions

Thank You.

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Well Posed Constraint-preserving Boundary Conditions For the Linearized Einstein Equations.
Comm. Math. Phys., 240:377âĂŞ395, 5 Sept. 2002.